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125

AN INVESTIGATION OF BASE PRESSURE RATIOS
OF BLUNT BASED-BODIES BY THE METHOD OF HYDRAULIC ANALOGY

A THESIS

Presented to

the Faculty of the Graduate Division
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In Partial Fulfillment
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Master of Science in Aeronautical Engineering

By

Jack Donald McAllister

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LIST OF SYMBOLS

x, y, z	spatial coordinates
u, v	velocity components of x, y
V	total velocity
a	speed of sound in air
M	Mach number
R_n	Reynolds number
W_n	Weber number
ρ	mass density
σ	surface tension
d	water depth
c	chord
h	base height
g	acceleration of gravity
ϕ	velocity potential
γ	ratio of specific heats
T	temperature
P	pressure

SUBSCRIPTS

x, y	partial derivative with respect to subscript
∞	free stream value
o	stagnation value
b	value at body base

TABLE OF CONTENTS

	Page
ACKNOWLEDGEMENTS	ii
LIST OF SYMBOLS	iii
LIST OF ILLUSTRATIONS	v
SUMMARY	vi
Chapter	
I. INTRODUCTION	1
II. THEORY	3
III. EQUIPMENT AND PROCEDURE	11
IV. RESULTS AND ANALYSIS	15
V. CONCLUSIONS	18
APPENDIX	19
BIBLIOGRAPHY	31

LIST OF ILLUSTRATIONS

Figure		Page
1.	General View of Georgia Tech Water Channel	20
2.	Dimensions of the Three Models	21
3.	Vertical Picture of Model Towed at Mach Number Two	22
4.	Vertical Picture of Model Base at Mach Number Two	22
5.	Flow Over Blunt-Based Body at Supersonic Velocity in Inviscous Fluid	23
6.	Flow Over Blunt-Based Body at Supersonic Velocity in Viscous Fluid	24
7.	Meniscus Effect at Model Base	25
8.	Center Line Survey of Base Pressure Ratio at Mach Number Two	26
9.	Transverse Survey of Base Pressure Ratio 0.2 Inches Behind Model Base at Mach Number Two	27
10.	Base Pressure Ratio Versus Mach Number for Three Model Lengths	28
11.	Base Pressure Ratio Versus $\frac{C}{h\sqrt{Re}}$ for Three Mach Numbers	29
12.	Base Pressure Ratio Versus $\frac{C}{h\sqrt{We}}$ for Three Mach Numbers	30

SUMMARY

Three geometrically similar blunt-based bodies of lengths two feet, one foot, and one-half foot were tested in the Georgia Tech water channel. The base pressure for each of these models was measured at Mach numbers of 1.5, 2.0 and 3.0. These measurements were compared with supersonic gas flow data for a geometrically similar body. Although the water channel data exhibited a trend similar to the trend for the same shape body in a viscous compressible gas, the quantitative agreement was not sufficient to allow prediction of base pressure ratios for bodies in supersonic viscous gas flow from results obtained in a water channel. The reason for this lack of quantitative agreement is that surface tension presents an additional factor which must be considered. This is reflected in the results by the dependency of the base pressure ratio on Weber number as well as Reynolds number. It is significant that the results of this investigation proved that an effect of viscosity exists in the water channel which is different from the effect of viscosity found in compressible gasses.

CHAPTER I

INTRODUCTION

When a supersonic flow separates from a body, there is a region of constant pressure behind the point of flow separation. For the case of a blunt-based body this flow separation logically occurs at the sharp corners of the base. The constant pressure behind the model base is called the base pressure. It has been observed that the base pressure is always much less than the free stream static pressure; this incomplete recovery of pressure results in an additional drag known as pressure drag. Because the total drag of blunt-based bodies is mostly pressure drag, the drag coefficient of a blunt-based body may be predicted from the base pressure.

The base pressure problem is essentially a mixing problem. The dead-air region behind the blunt base must mix with the supersonic free stream through a layer that is the boundary layer detached from the body. This complex flow mechanism has been investigated experimentally by Chapman, Ref. (1), by the use of a supersonic wind tunnel. The present investigation is concerned with testing one particular model shape used by Chapman. Three models of lengths of one half, one, and two feet were tested at supersonic velocities in the Georgia Tech water channel.

Theoretical work on the analogy between a compressible gas and the flow of water with a free surface was first presented by Riabouchisky, Ref. (2), in 1932. The validity of the hydraulic analogy was conclu-

sively proved by Preiswerk, Ref. (3). The most recent advances in theory and experimental applications are those of Crossley and Harleman, Ref. (4).

Work with the analogy at the Daniel Guggenheim School of Aeronautics was commenced in 1948 by John Hatch, Ref. (5). Under his supervision, a water channel was constructed and preliminary tests were conducted which demonstrated considerable possibilities. In 1949 John Catchpole, Ref. (6), conducted further tests and altered the equipment somewhat. The method presently used to determine the water height was tested and shown to be suitable in 1950 by Dallas Ryle, Ref. (7). The previous work has been concerned with systems for which viscosity was not a predominant factor. The present set of experiments were chosen to test the effect of viscosity upon water table results because of the great influence of viscosity upon base pressure ratios in air.

CHAPTER II

THEORY

The theory of the hydraulic analogy as presented by Preiswerk, Ref. (3), will be reviewed here. The analogy is best demonstrated by comparing the important governing equations for each flow.

The equation of continuity is for the water table

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$$

and for two dimensional gas flow

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$$

From these two equations we see that the analogy is:

$$d/d_0 = \rho/\rho_0 \quad (1)$$

The energy equation gives for the water table

$$V^2 = 2g(d_0 - d)$$

$$V_{max} = \sqrt{2gd_0}$$

and for two dimensional gas flow

$$V^2 = 2gC_p(T_0 - T)$$

$$V_{max} = \sqrt{2C_pT_0}$$

Equating the ratio of V/V_{\max} for the two types of flow:

$$\text{or} \quad \frac{d_0 - d}{d_0} = \frac{T_0 - T}{T_0} \quad (2)$$

$$\frac{d}{d_0} = \frac{T}{T_0}$$

Considering (1) and (2) the relation between density and temperature is

$$\frac{\rho}{\rho_0} = \frac{T}{T_0} \quad (3)$$

For adiabatic gas flow the well known temperature density relation is

$$\frac{\rho}{\rho_0} = \left(\frac{T}{T_0} \right)^{\frac{1}{\gamma-1}} \quad (4)$$

Equations (3) and (4) are compatible only if $\gamma = 2$. In the analogy given by Preiswerk, the flow in a water channel is comparable to adiabatic gas flow with $\gamma = 2$.

The differential equation of the velocity potential for the water table is

$$\phi_{xx} \left(1 - \frac{\phi_x^2}{g\alpha} \right) + \phi_{yy} \left(1 - \frac{\phi_y^2}{g\alpha} \right) - 2\phi_{xy} \frac{\phi_x \phi_y}{g\alpha} = 0$$

and for two dimensional gas flow is

$$\phi_{xx} \left(1 - \frac{\phi_x^2}{a^2} \right) + \phi_{yy} \left(1 - \frac{\phi_y^2}{a^2} \right) - 2\phi_{xy} \frac{\phi_x \phi_y}{a^2} = 0$$

A comparison of the previous two equations yields

$$\frac{gd}{2gd_0} = \frac{a^2}{2gh_0} \quad (5)$$

The basic wave propagation velocity, \sqrt{gd} , as given by Page, Ref. (8), corresponds to the pressure propagation velocity or sound velocity in gas flow. In water flowing at velocities greater than \sqrt{gd} the velocity may strongly decrease and the depth increase when the flow is disturbed. An unsteady motion of this type is called a hydraulic jump and corresponds to a shock wave in a gas.

Crossley and Harleman, Ref. (4), demonstrate that the only applicable analogies of those presented above are the comparison of equations of continuity and velocity potential. The density ratios may be determined by equation (1), and then pressure ratios are determined from the adiabatic relationship

$$\frac{P}{P_0} = \left(\frac{\rho}{\rho_0} \right)^\gamma \quad (6)$$

Then the base pressure ratio is

$$\frac{P_b}{P_\infty} = \left(\frac{d_b}{d_\infty} \right)^\gamma \quad (7)$$

The logical first case in considering the flow about a blunt-based body is that of an inviscous supersonic flow. The development of Chapman, Ref. (9), will be briefly reviewed here. Fig. 5 is a schematic of the base flow mechanism for this case. It is evident that no one

flow is determined for this case. The flow may continue undisturbed, it may be deflected toward the center line by a Prandtl-Meyer expansion fan or it may be deflected away from the center line by a shock wave. For this case there are thus an infinity of flow regimes limited only by the maximum angles of flow deflection in the two directions. Some mechanism must exist which singularly determines the base flow. It will now be shown that the viscosity of the fluid is the property which governs the flow at the base because of a boundary layer on the body.

Fig. 6 is a schematic of the base pressure flow mechanism occurring at the base of the blunt-based body in a viscous supersonic stream. As a consequence of the viscosity a boundary layer is formed on the surface of the body. The boundary layer is turned at the base by the Prandtl-Meyer expansion fan in the same manner as a perfect fluid. This layer becomes a free jet in an extremely short distance downstream of the base. The theory of Lees and Crocco, Ref. (10), predicts that this distance is of the order of one boundary layer thickness. As the free jet merges with the one from the other surface, it is turned back toward the center line by the trailing shock and ultimately forms the wake. The mass removed from the base by the wake must be replaced with mass from the external flow. This external mass is brought into the base area or dissipative flow area by a mixing between the boundary layer and the external flow. Crocco shows that a critical point or wake throat exists which maintains the balance between the flow in and the flow out. The base pressure is controlled by the boundary layer through the mixing process. The fact that Chapman, Ref. (1), was able to correlate experimental data by using the ratio of boundary layer thickness at the base to base thick-

ness emphasises the importance of the role the boundary layer plays in determining base pressure. Since the boundary layer thickness for laminar flow is proportional to the distance from the leading edge divided by the square root of Reynolds number, Chapman chose for the applicable non-dimensional parameter the chord divided by the base height times the square root of Reynolds number $c/h\sqrt{Re}$. His results for an airfoil similar to the one used in these tests are given in Fig. 11.

The base pressure theory of Lees and Crocco, Ref. (10), which takes into account the effect of fluid viscosity, is highly complex, lengthy, and of qualitative value only. For these reasons, the theory will not be presented here, but a summary of the fluid-mechanical explanation of the base pressure problem based on this theoretical work will be given in its place. The problem is divided into four flow regimes which in the order of increasing Reynolds number are:

- (1) At sufficiently low Reynolds numbers the flow is laminar in the boundary layer and in the wake throughout the portion in which the majority of the recompression occurs. As Reynolds number increases, the laminar mixing rate decreases and the base pressure increases slowly. The base pressure is relatively high because of the low laminar mixing rate.
- (2) As Reynolds number increases, transition moves upstream in the wake increasing the local mixing rate by a large amount (the turbulent mixing rate is approximately five to ten times as large as the laminar mixing rate) causing the base pressure to drop very rapidly with increasing Reynolds number. After transition has moved close to the airfoil base,

the base pressure ratio continues to drop with increasing Reynolds number, as long as the airfoil boundary layer remains laminar because of the decreasing boundary layer thickness. If the boundary layer transition is at sufficiently high Reynolds number and if the ratio of chord to airfoil trailing edge thickness is low, the base pressure ratio depends only on the ratio of boundary layer thickness to trailing edge thickness over a considerable Reynolds number range in this regime.

(3) As the transition moves forward in the airfoil boundary layer with increasing Reynolds number, the base pressure ratio first increases due to a thickening of the boundary layer at the base. As transition moves further upstream the normal decrease in local turbulent boundary layer thickness with increasing Reynolds number offsets the thickening effect and the base pressure begins to decrease again.

(4) At high Reynolds numbers transition is essentially fixed and the base pressure ratio decreases slowly but noticeable with increasing Reynolds number, because of the decrease in turbulent boundary layer thickness. Over this range of Reynolds numbers, the base pressure ratio is a function of the parameter Reynolds number to the one-fifth power times base height divided by chord or some similar parameter involving the product of the ratio of base height to chord and a logarithmic function of Reynolds number.

In considering the base pressure problem utilizing the hydraulic analogy of the water channel it appears that the Weber number as well as the Reynolds number may be an important parameter. Reynolds number, which is governed by the ratio of inertia forces to friction forces, has already been shown to be important through its direct connection with

the boundary layer mixing rate. Weber number is governed by the ratio of inertia forces to surface tension forces. A brief derivation of Reynolds number and Weber number will be used to show where these non-dimensional parameters arise.

The X component of the Navier-Stokes equation is:

$$\rho \frac{\partial u}{\partial t} = \rho X + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}$$

We wish to determine the units of the ratio of the inertia forces to shear forces.

$$\text{Ratio} = \frac{\rho \frac{\partial u}{\partial t}}{\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}} \sim \frac{\rho u \frac{\partial u}{\partial x}}{\mu \frac{\partial^2 u}{\partial x^2}}$$

The ratio has the units of

$$\left| \frac{\rho V \frac{V}{L}}{\mu \frac{V}{L^2}} \right| = \left| \frac{\rho V L}{\mu} \right|$$

Thus the ratio of inertia forces to shear forces gives the Reynolds number. Similarly for the units of the ratio of inertia forces to surface tension forces one obtains

$$\text{ratio} = \left| \frac{\rho V \frac{V}{L}}{\frac{\sigma}{L^2}} \right| = \left| \frac{\rho V^2 L}{\sigma} \right| = \text{Weber number}$$

In the water channel the large pressure gradients through the free wake of the base are represented by large gradients in water depth. As the model is towed through the water, it causes these large depth gradients to be formed in a short period of time, this can only mean that the surface area is changing rapidly. Surface tension is normally represented as force per unit length. However, its units may be represented in another way as energy per unit area (it is the energy required to form a unit surface area). Because of the rapid change of surface area in the wake region surface tension becomes an important factor in controlling the wake phenomena. The Weber number should then have an effect on the results.

As the base pressure is controlled in great part by the viscosity and surface tension, comparison of the results in air and water table for base pressure ratio should bring out the relative importance of viscosity and surface tension upon the water table results.

CHAPTER III

EQUIPMENT AND PROCEEDURE

The Georgia Tech water channel is of the type in which the model is moved through static water. The main advantage of this type is the relative small expense compared to one in which the model is stationary with the water flowing past it. There is no need to consider a potential energy term (ρZ) in the energy equation when the model is towed through the water, where the term must be considered when the water flows past the model, and there are no transient shocks or undesirable effects at the test section entrance present in the table with flowing water. Other advantages of this type include easy acceleration of the flow, simple construction, and the absence of boundary layer effects from the channel bottom. The main disadvantage is the difficulty of measuring the water depth along the model.

A general view of the water channel is shown in Fig. 1. The frame is of bolted structural steel supporting a channel four feet wide, twenty feet long, and approximately one and one-fourth inches deep. The bottom of the channel is of plate glass in two five foot sections and one ten foot section, the ten foot section being the center or test section. The transverse steel members are spaced at thirty inch intervals and are supported by screw jacks enabling the glass surface over which the model slides to be leveled. A drain is provided at one end of the channel in order to change the liquid.

The model carriage is of welded steel tubing construction. It is moved along the channel on four rubber wheels which transfer the weight of the carriage to the upper horizontal steel members of the frame which also serve as rails. Four rubber wheels with vertical axes are located at the carriage frame corners to prevent any sidewise motion of the carriage. The model is supported ahead of the carriage by a vertical free acting mount producing the towing force and permitting the model weight to act on the channel bottom. Safety stops are placed at the ends of the carriage track to prevent overrunning of carriage and model. The carriage is driven by a one quarter horsepower, single phase, alternating current electric motor through a $3/32$ inch continuous steel cable. A reversing mechanism and a "Speed Ranger" device is used to control the motion in either direction and at varied speeds.

Timing for accurate speed adjustment is accomplished by means of a microswitch placed on the track. A cam 2.911 feet in length is attached to the carriage so that it will close the microswitch and operate an electric timer. The timer is located with the instruments and switches for starting, reversing and operating the drive mechanism on the control panel above the water channel.

The model base is fitted with a plexiglass bracket from which is suspended a steel needle probe. This probe is attached to an adjustable screw which is screwed into the plexiglass bracket. The bracket for the largest model is adjustable so that a complete survey of the water height in the area of the model base may be made. Contact of the probe with the water completes the grid circuit of a vacuum tube causing a relay to operate a signal light. The model is towed through the water and the probe is

adjusted vertically until it just touches the water. The status of the signal light determines if the probe is in contact with the water. The water depth is measured to an accuracy of 0.001 inch by means of a height gage with the model on a surface plate.

The static depth of the water channel is determined by setting the probe to the desired depth and filling the channel to that depth using the signal device. The point at which the model is placed to fill the channel is used as the test point for measurements with the signal device.

Fig. 2 shows the dimensions of the three models used for the tests. The largest model was constructed of aluminum; the two other models were constructed of mahogany. This model shape is designated as 10-0.75 by Chapman, Ref. (1).

A survey of the base pressure was made along the model center-line and across the model two-tenths of an inch behind the base at a Mach number of two using the two foot model. An investigation of the static meniscus was made to be used as a correction for meniscus effect when the model is being towed. The results of this survey, given in Fig. 7, show that measurements made nearer to the model than two-tenths of an inch are effected by the meniscus. The probability of the meniscus being identical for the case of the model stationary and the case of the model being towed through the water is undoubtedly slight but it is felt that the use of a correction obtained from static measurements will, to a large extent, correct for this effect.

The base pressure ratio was determined at Mach numbers of 1.5, 2.0 and 3.0 for each of the three models. The temperature was recorded

for each run so that the kinematic viscosity and surface tension of the water could be determined.

CHAPTER IV

RESULTS AND ANALYSIS

Fig. 3, a vertical picture of the model at Mach number two, shows the similarity between water channel wave configurations and well known schlieren photographs of blunt-based models at supersonic velocities in air. The shock waves at the leading edge and in the wake are represented by hydraulic jumps. Fig. 4 is a typical photograph of the flow at the model base at Mach number two. This photograph demonstrates the similarity between the water channel and compressible gas flow for a blunt-based body; see Fig. 6 for a schematic presentation of this flow for a viscous compressible gas.

The results of a centerline survey of base pressure ratio conducted with the two foot model at a Mach number of two is given in Fig. 8. The pressure ratio is constant out to a distance of 0.4 inches and then increases as recompression begins in the neighborhood of the trailing shock wave. Fig. 9, a cross survey of base pressure of the same model at Mach number two taken 2/10 inch behind the base, demonstrates that the base pressure is constant across the base and that very large gradients are formed at the boundaries of the base pressure area substantiating the argument for surface tension effect previously stated. Figs. 8 and 9 conclusively demonstrate that a base pressure may be produced by the water table analogy for supersonic flow. The fact that the base pressure was different for the two surveys although the pres-

sure was constant in each survey, is explained by noting that the water was fresh for the center line survey but it was stale and somewhat contaminated when the cross survey was conducted. This also substantiates the theory that surface tension is a factor in determining the base pressure. It was decided that the water should be changed just before each set of runs to insure consistent results.

Fig. 10 is a presentation of the base pressure ratios for the three models. The ratios are qualitatively similar to gas flow data in that the base pressure decreases as Mach number increases and there is a decided scale effect. The remainder of the analysis was undertaken in search of the similarity parameter that applies to this particular problem.

The base pressure data are compared with the results of Chapman's investigation, Ref. (1), in Fig. 11. These data are compared on the basis of the parameter chord divided by base height times the square root of Reynolds number. The water table data show reasonable qualitative agreement but it is obvious that the water table data could not be used to predict the base pressure occurring in compressible gas flow. For Fig. 12 Weber number was substituted for Reynolds number in the similarity parameter used by Chapman. This parameter is not of the same order of magnitude as Reynolds number hence the two sets of data can not be compared exactly. From Figs. 11 and 12 it may be concluded that neither Reynolds number nor Weber number is the singular governing similarity parameter but the parameter is a function of both of these with Reynolds number being by far the most important.

An attempt was made to fit the test data for the water channel

to the data obtained by Chapman by the use of the parameter $\frac{c}{5TK^n} A(\omega_h)^n$ where A and n are constants used to fit the curve. It was found that for a Mach number of 1.5, A is $10^{14.6}$ and n is -1.84; while for a Mach number of 2.0, A is $10^{-3.25}$ and n is 0.328. The only conclusion which may be made from this calculation is that no one simple combination of Reynolds number and Weber number may be used as a similarity parameter over a range of Mach numbers.

CHAPTER V

CONCLUSIONS

The water channel analogy for supersonic gas flow is qualitatively correct for the case of blunt-based bodies but the quantitative agreement is not sufficient to justify using water channel data to predict the base pressure ratios for supersonic gas flow. The reason for the lack of quantitative agreement is that surface tension presents an additional factor which must be considered. This is reflected in the results by the dependency of the base pressure ratio on Weber number as well as Reynolds number. The wave configuration about the blunt base is identical for the two types of flow, thus the water channel may be used to demonstrate the flow patterns about blunt-based bodies at supersonic velocities.

In all future water channel work involving accurate measurement of water depth near the test model, a correction must be made for the meniscus existing at the model. A correction based on static depth measurements near the model yields satisfactory results.

APPENDIX

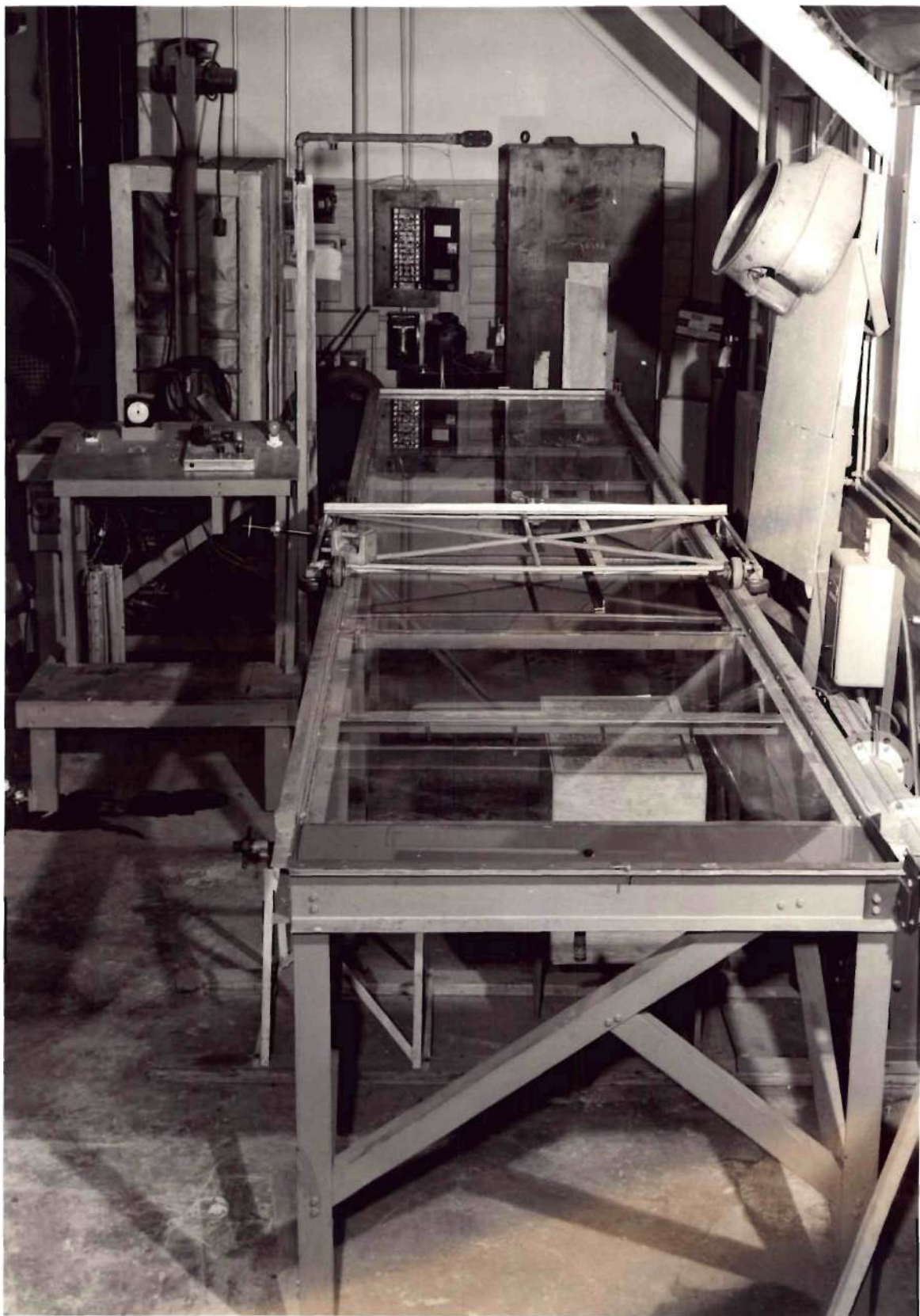
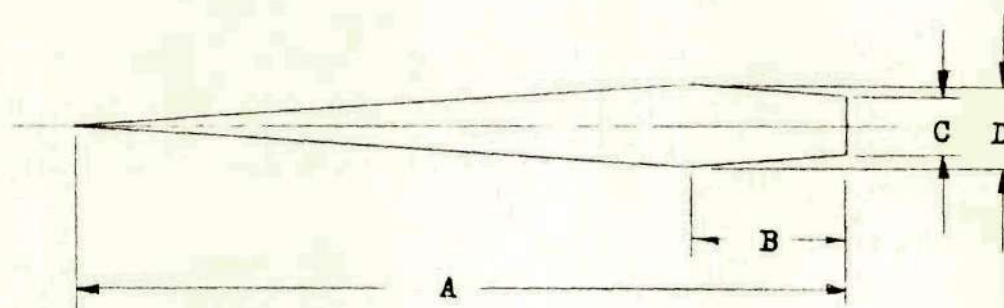


Fig. 1 General View of Georgia Tech Water Channel



A	B	C	D
6.00	1.20	0.46	0.60
12.00	2.40	0.90	1.20
24.00	4.80	1.80	2.40

Figure 2

Dimensions of the Three Models

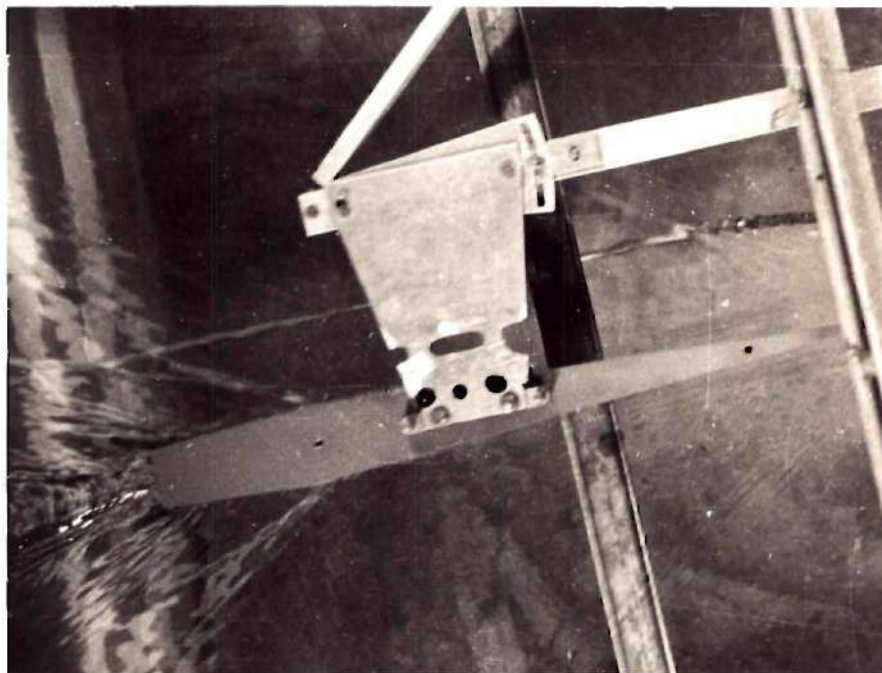


Fig. 3 Model Towed at Mach Number Two

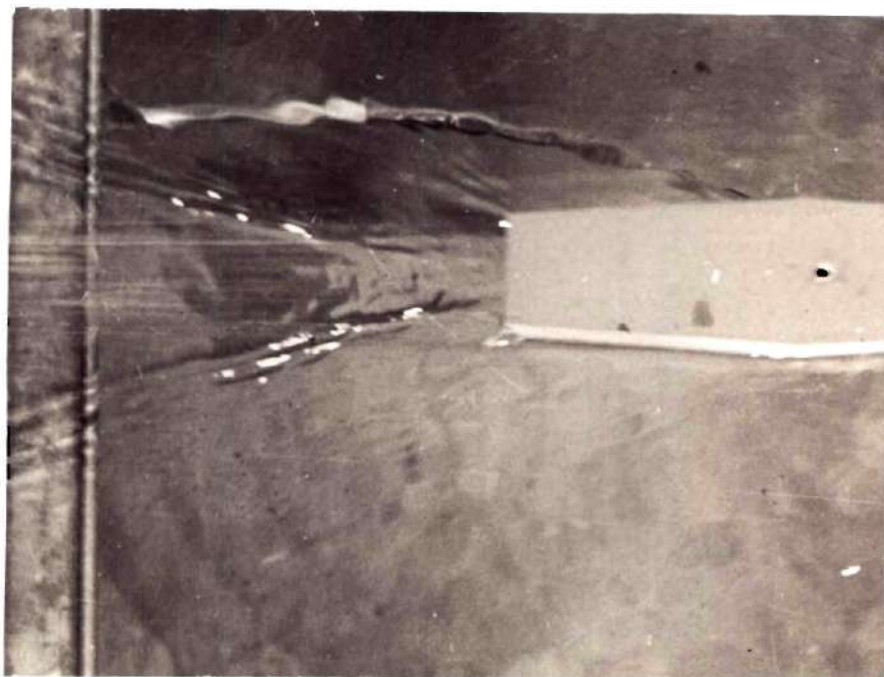


Fig. 4 Model Base at Mach Number Two

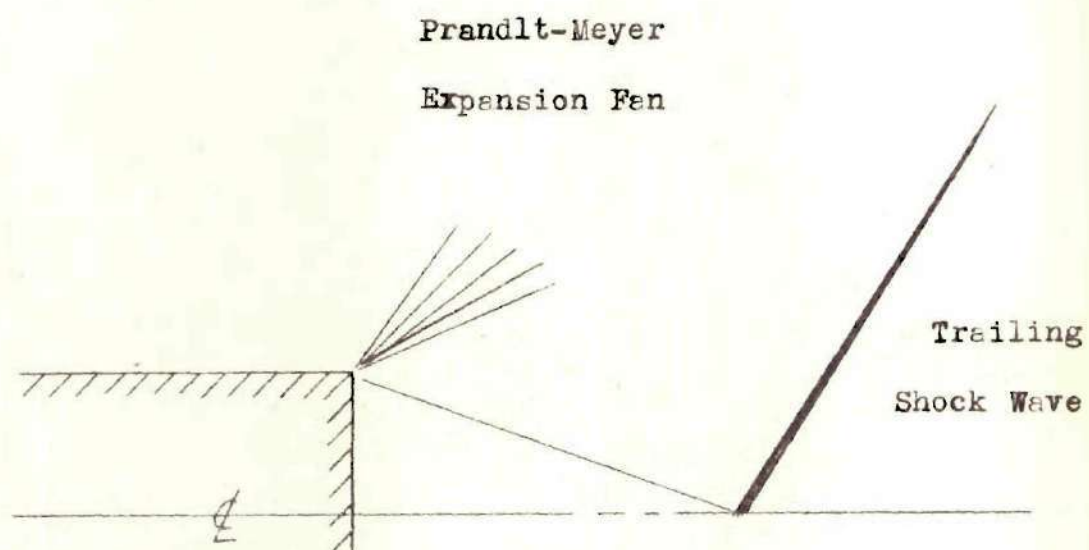


Figure 5
Flow Over Blunt Base Body at Supersonic Velocity
in Inviscous Fluid

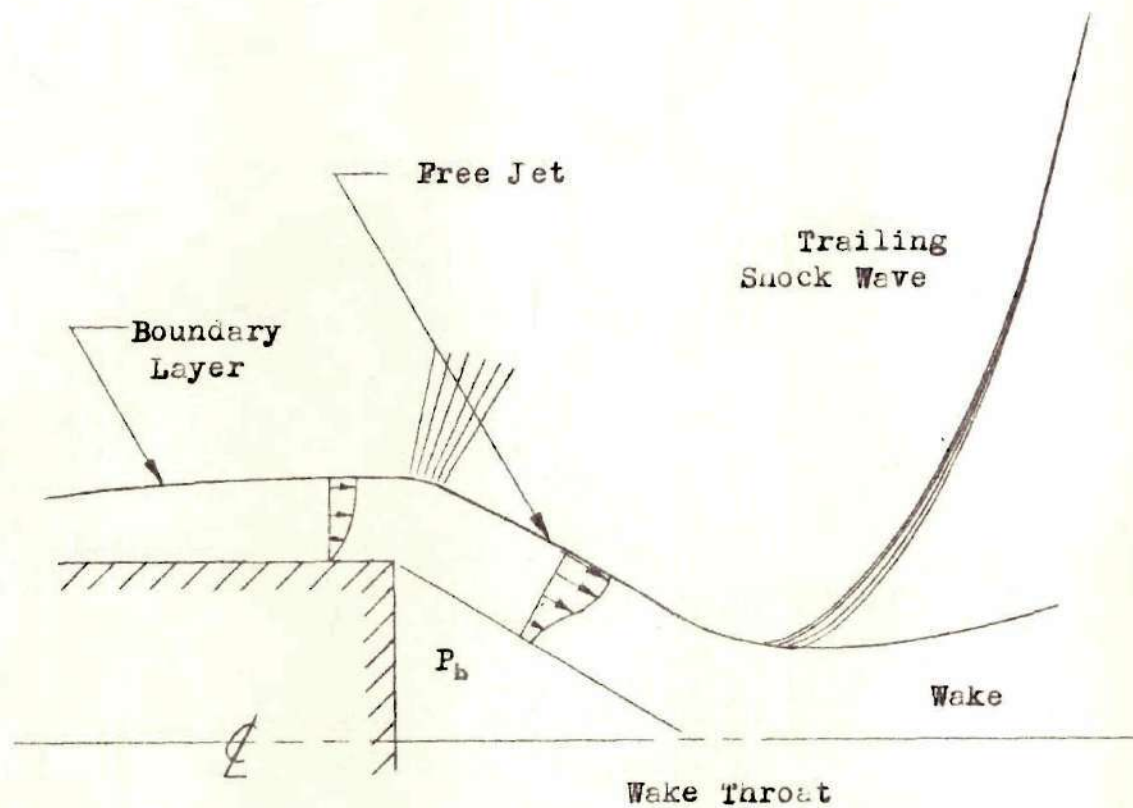


Figure 6
Flow Over Blunt-Base Body at Supersonic
Velocity in Viscous Fluid

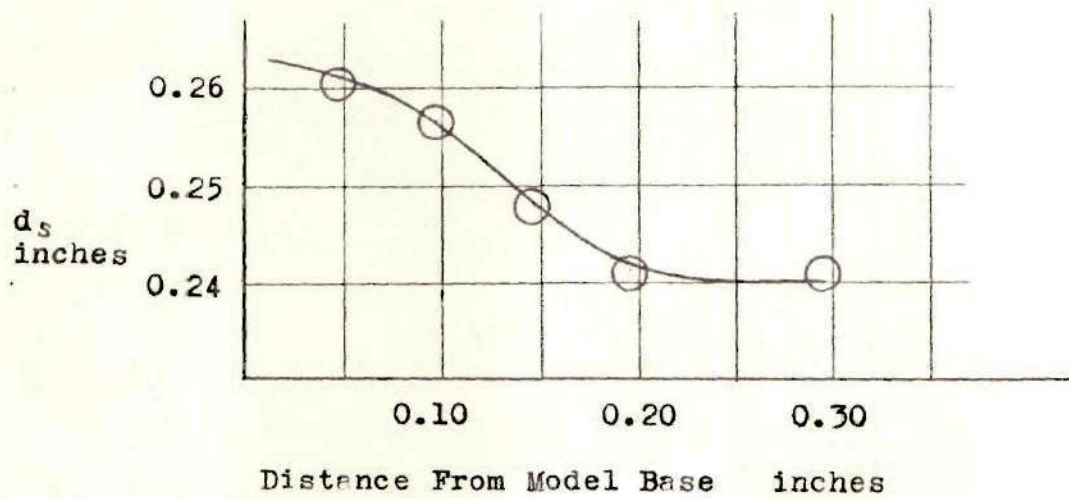


Figure 7

Meniscus Effect at Model Base

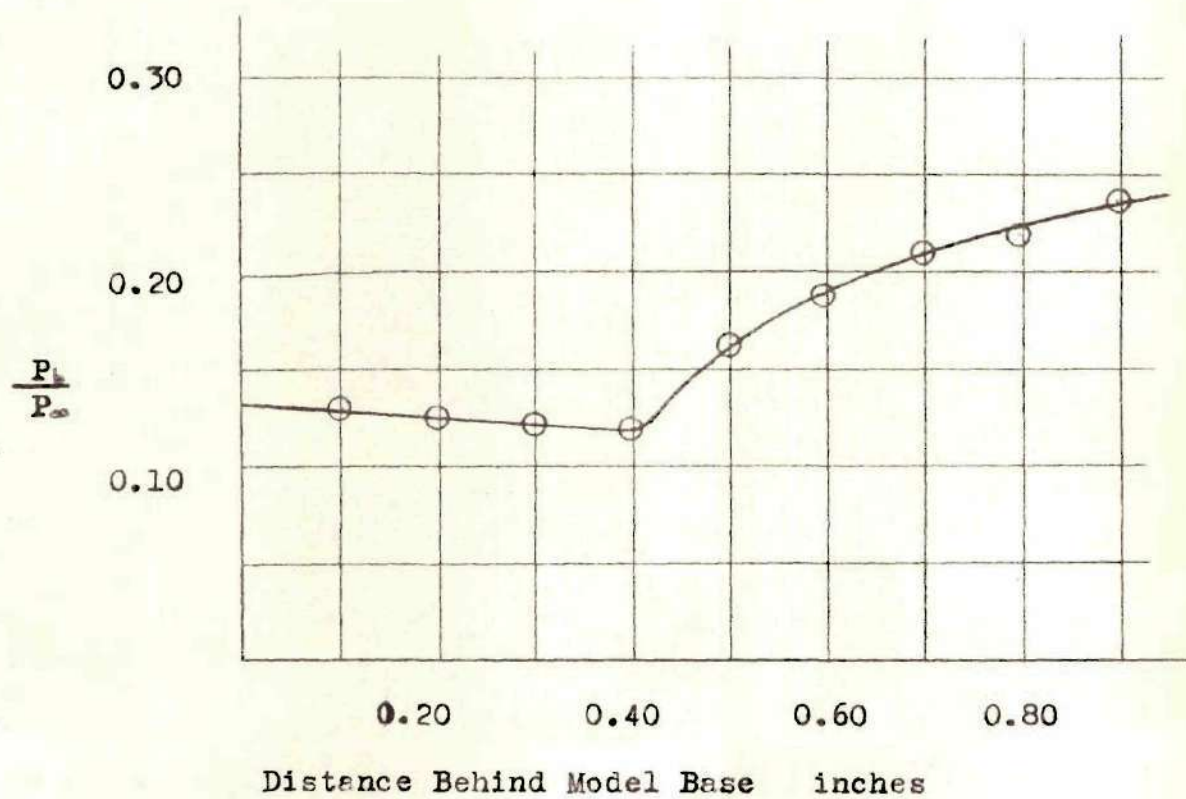


Figure 8

Center Line Survey of Base Pressure Ratio

at Mach Number Two

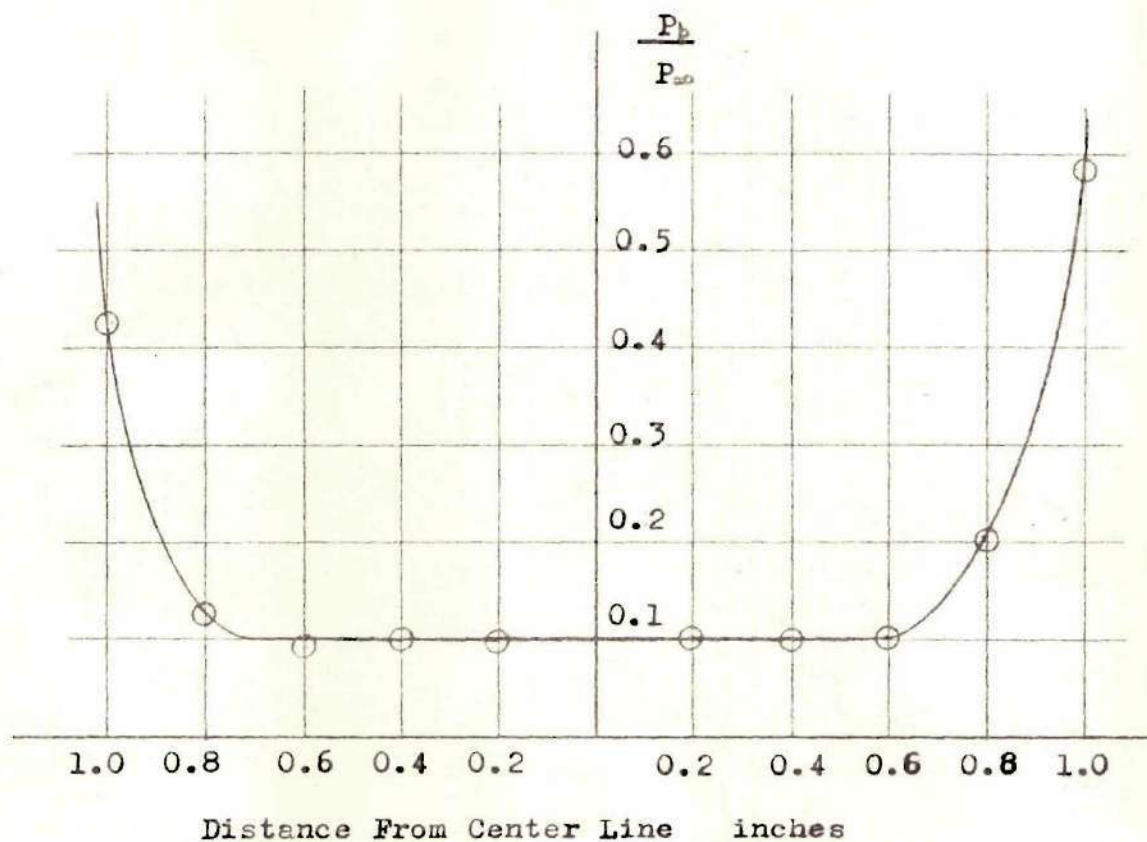


Figure 9

Transverse Survey of Base Pressure Ratio

0.2 Inches Behind Model Base at

Mach Number Two

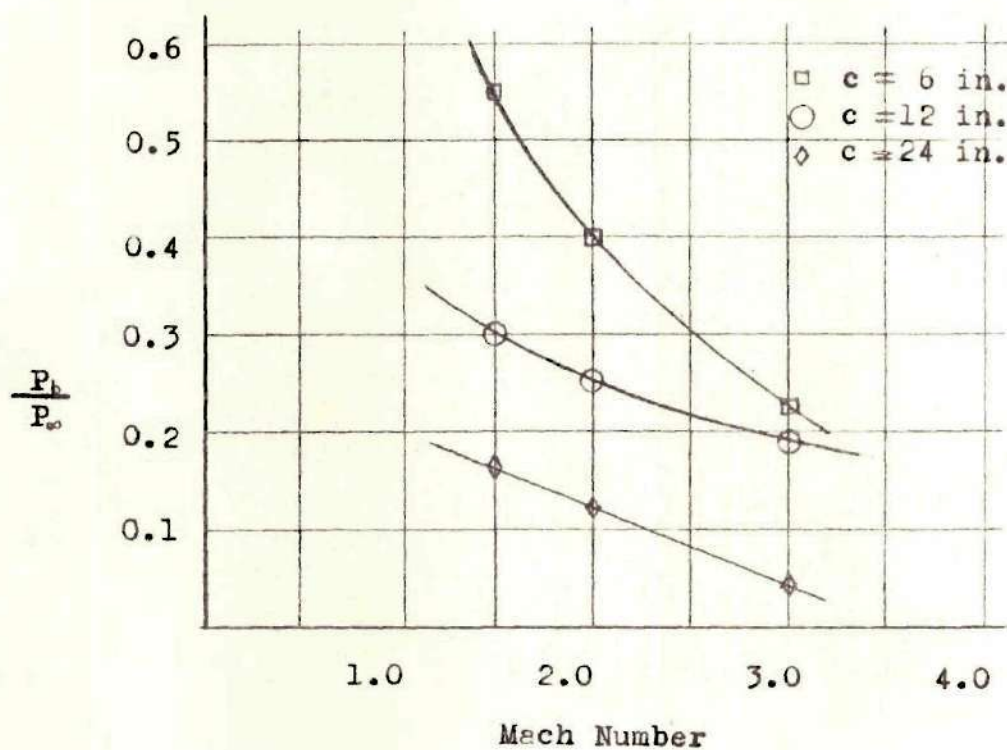


Figure 10
Base Pressure Ratio Versus Mach Number
for Three Model Lengths

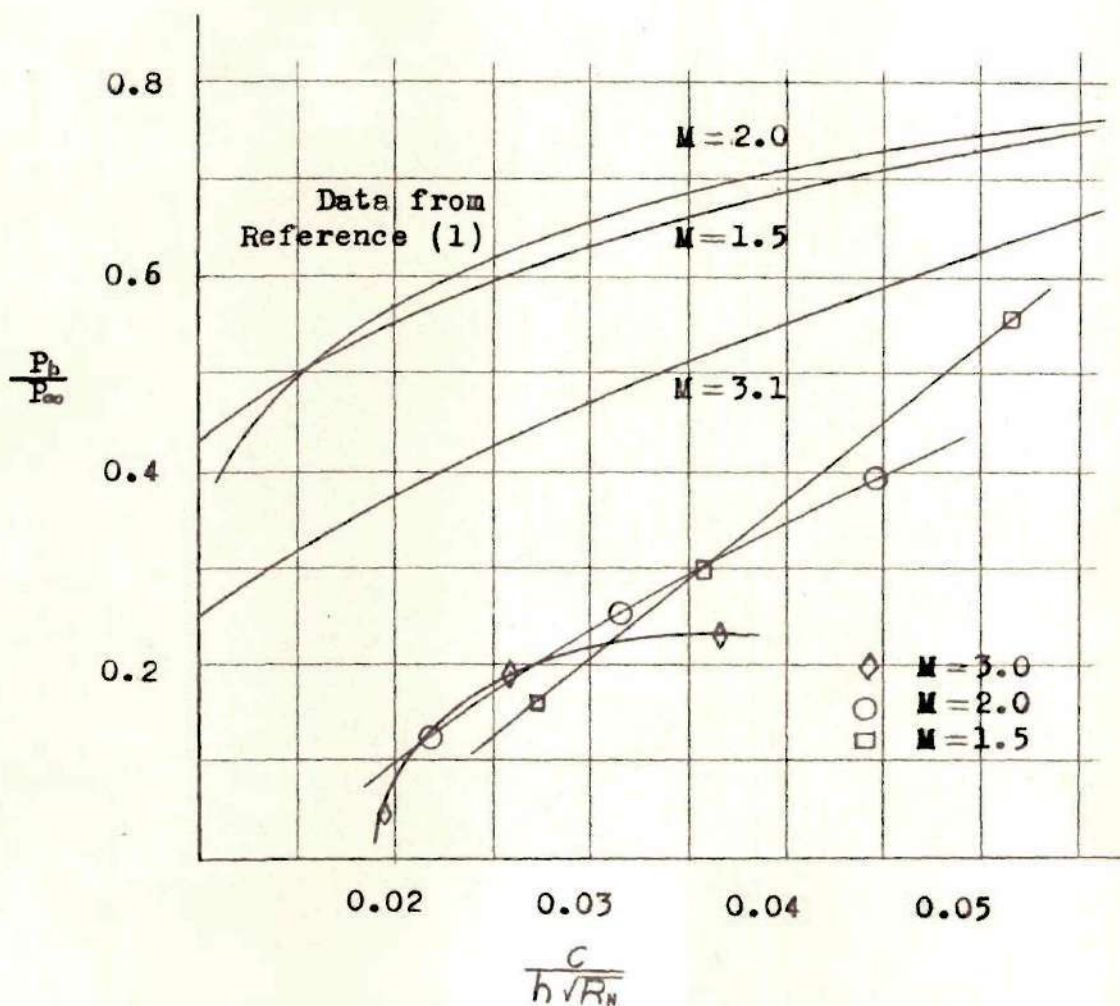


Figure 11

Base Pressure Ratio Versus
for Three Mach Numbers

$$\frac{C}{h\sqrt{R_N}}$$

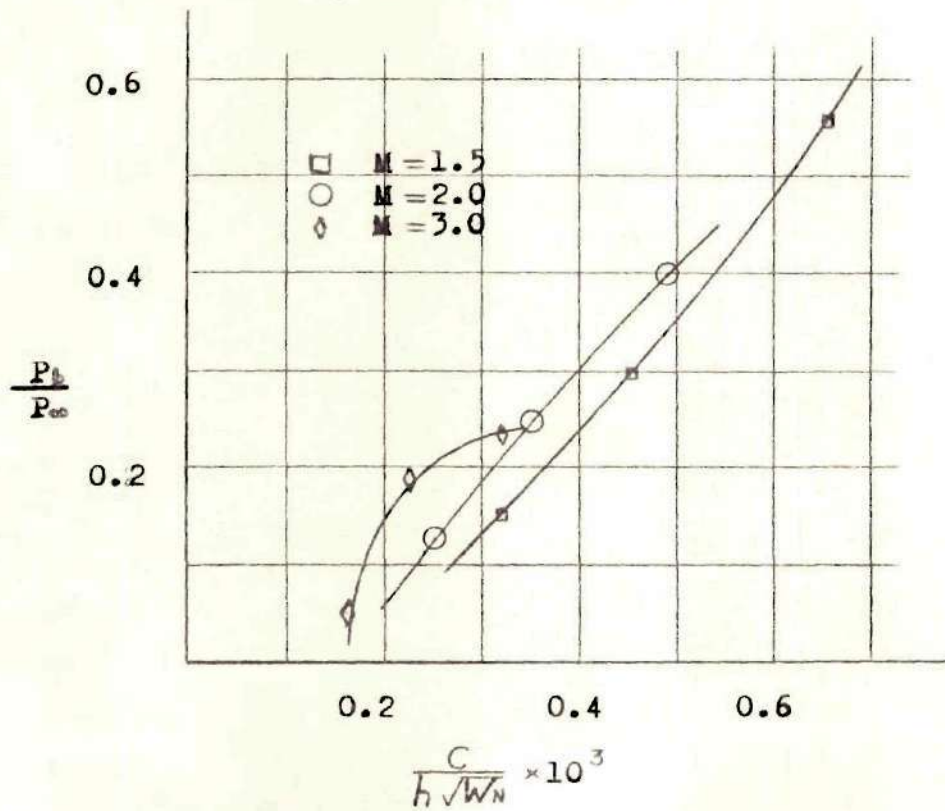


Figure 12

Base Pressure Ratio Versus $\frac{C}{h\sqrt{W_N}}$
for Three Mach Numbers

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